On an Extended PCAC Relation

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We explore a consistent way to extend the partially conserved axial vector current (PCAC) relation and corresponding current algebra results in two strongly correlated directions: 1) towards a search for a set of systematic rules for the establishment of PCAC related relations in a finite low momentum transfer region, and, for the extrapolation of the momentum transfer q^2 to zero when deriving the low energy PCAC results that can be compared to experimental data and 2) towards taking into account, besides the conventional one, the only other possibility of the spontaneous chiral symmetry breaking, $SU(2)_L \times SU(2)_R \to SU(2)_V$, inside a baryonic system by a condensation (in the sense to be specified in the paper) of diquarks. The paper includes investigations of a chiral Ward-Takahashi identity, the explicit chiral symmetry breaking by a finite current quark mass, the modification of the PCAC relation and its consequences. Two explicit relations between the nucleon axial vector form factor $g_A(q^2)$, pseudo-scalar form factor $g_P(q^2)$ and the pion-nucleon coupling constant $g_{\pi NN}(q^2)$ is obtained. One of the relation is confirmed, within the experimental error, by observation in the $0 < -q^2 < 0.2$ GeV^2 region. The other one, which relates $g_A(q^2)$ and $f_\pi(q^2) g_{\pi NN}(q^2)$ is studied by using known empirical facts and dispersion relation. Theoretical uncertainties are discussed. Certain inconsistencies, which is in favor of the introduction of diquark condensation, is discovered. We briefly discuss how diquark condensation could provide an answer to the question of where about of the quark numbers in a nucleon and a nucleus, which is raised in explaining puzzles observed in the violation of Gottfried sum rule and EMC effects.

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I. INTRODUCTION

Model independent partially conserved axial vector current (PCAC) relation and corresponding current algebra results are in conformity with experimental data within a few percent. It is quite impressive compared to other observables in strong interaction. This good agreement between theory and experiments is interpreted as due to an underlying approximate chiral $SU(2)_L \times SU(2)_R$ symmetry, which is explicitly broken down by up and down current quark masses of a few MeV, much smaller than the hadronic mass scale of 1 GeV. The lightest hadronic particle pions are considered as, in the limit that the current masses of the up and down quarks vanish, the Goldstone bosons of a spontaneous breaking down of the above mentioned symmetry induced by a non-vanishing vacuum expectation value of the quark field bilinear operator $\bar\psi\psi$, namely, $\langle 0 |\bar\psi\psi| 0\rangle \neq 0$. Equipped with new relevant empirical information, we provide a refined analysis of the old results, which are based on an extrapolation of the physical quantities to $q^2 \to 0$ [1], to include a wider range of momentum transfer, and, may be more interestingly, to explore possible extensions that are observable and are nevertheless consistent with our present knowledge.

The possibility of the formation of a superconducting phase in a massless fermionic system, which is a realization of one of the possible phases in which the chiral symmetry is spontaneously broken down to a flavor (isospin) symmetry, is investigated in Ref. [2] base on a 4-fermion interaction model. It was argued that the relativistic superconducting phase might be relevant to the creation of baryons in the early universe due to its quantum mechanical nature. If this scenario reflects the nature at least at a qualitative level, it would be of interest to study the possible existence of such a phase in the hadronic system at the present day condition. Since it is unlikely that the present day strong interaction vacuum at large scale is in the superconducting phase for reasons given in the following, the only regions where the superconducting phase can be found are inside a nucleon, a nucleus, the center region of a heavy ion collision, and an astronomical object like a neutron star, a quasar, etc. Albeit the possible superconducting phase in the baryon

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creation era of the early universe may have had disappeared entirely at certain previous time during the evolution of the universe, it is still a worthwhile effort to search for such a phase at the present time. One of the reasons is that if such a phase can be found, its properties can be studied in domestic laboratories. Its existence is a reasonable possibility since it is shown [2] that given suitable coupling constants, the phase in which $\langle 0 | \bar{\psi}\psi | 0 \rangle \neq 0$ changes into a superconducting phase as the baryon (or quark) density is raised. The empirical need for such an assumption will be discussed in the conclusion parts of the paper rather than in this section because most of the individual phenomenon considered has its own explanation in terms of conventional picture with various degrees of success at the present theoretical and/or experimental precisions. We are interested in a search for a consistent explanation of a large set of observations. Motivated by these considerations, we explore some of the phenomenological consequences of a possibility in which the interior of a nucleon contains diquark condensation [3], which is enhanced in a nucleus, that spontaneously breaks the chiral symmetry.

In order to differentiate the two hidden chiral (asymmetric) phases, the phase in which $\langle \bar{\psi}\psi \rangle \neq 0$ and no diquark condensation is called the Nambu phase [4] and the phase in which there is a diquark condensation is called the superconducting phase in this paper. The model dependency of the discussion is reduced as much as possible. For that purpose, we classify the spontaneous chiral symmetry belonging to the same chain, namely, $SU(2)_L \times SU(2)_R \to SU(2)_V$, into categories characterized by their order parameters introduced in Ref. [2], which are generic in nature. The Nambu phase is characterized by a non-vanishing σ and vanishing $\phi^{c\mu}$ and $\bar{\phi}^{\mu}_{c}$. The superconducting phase is characterized by non-vanishing $\phi^{c\mu}$ and $\bar{\phi}^{\mu}_{c}$ and possibly a non-vanishing σ. The generality of the discussion allows the results to be applied to any model with a similar phase structure as the one obtained in Ref. [2].

The paper is organized in the following way. In section II, a chiral Ward-Takahashi identity is studied by presenting the main results, which establish the existence of the Goldstone diquark excitation in the superconducting phase. Section III deals with the perturbation of a small current quark mass term in the Lagrangian. The mixing of the Goldstone diquark excitation in the divergence of the hadronic axial vector current operator in a baryonic system containing diquark condensation is demonstrated. Under the assumption that there is a diquark condensation inside a nucleon, we investigate in section IV the necessity and the consequences of a modification of the PCAC relation and related current algebra results. The theoretical uncertainties and the content of two assumptions introduced in section IV is discussed in section V. In section VI, the empirical basis for our extension of PCAC relation is discussed. We find certain inconsistence that suggests the need for a consideration of diquark condensation or an extension of the PCAC relation. Section VII is devoted to other observations that are considered to favor the introduction of diquark condensation with different degrees of certainty. We demonstrate the need for a further study of that possibility with better theoretical and experimental precision . Section VIII contains a summary.

II. THE CHIRAL $SU(2)_L \times SU(2)_R$ WARD-TAKAHASHI IDENTITY

We study a chiral Ward-Takahashi identity in this section to show the existence and the basic properties of the Goldstone diquark mode in the superconducting phase. One way of studying the properties of pions and Goldstone diquarks is to solve certain Bethe-Selpeter equation provided that an explicit model, which is not important for the study of this paper, is specified. The method used here depends upon no particular model assumptions but the structure of the quark self energy. In order to investigate the superconducting phase, an 8-component "real" representation for the quark field Ψ is introduced [2]

$$\Psi = \begin{pmatrix} \psi \\ \tilde{\psi} \end{pmatrix} \tag{2.1}$$

with ψ and $\tilde{\psi}$ 4-component Dirac spinors. The "real" condition for Ψ is

$$\bar{\Psi} \equiv \left(\psi^\dagger \gamma^0, \tilde{\psi}^\dagger \gamma^0 \right) = \Psi^T \Omega, \tag{2.2}$$

where

$$\Omega \equiv \begin{pmatrix} 0 & C^{-1} i\tau_2 \\ C i\tau_2 & 0 \end{pmatrix}, \tag{2.3}$$

with superscript "T" denoting transpose, C the charge conjugation operator and τ_2 the second Pauli matrices acting on the flavor components of Ψ. The 8-component spinor Ψ for quarks given by Eq. 2.1 is used through out the paper.

The axial vector current vertex $iA^{5a}_{\mu}(p', p)$ between single quark states is written as

$$iA_\mu^{5a}(p+\frac{q}{2},p-\frac{q}{2}) = \frac{i}{4}\gamma_\mu\gamma^5\tau^a O_3 + \Gamma_\mu^{5a}(p+\frac{q}{2},p-\frac{q}{2}), \tag{2.4}$$

where Γ_μ^{5q} is the radiative part of A_μ^{5a}, the initial and final state (8-component) quark spinors are suppressed and q_μ stands for the 4-momentum transfer. Here O_3 and $O_{(\pm)}$ in the following are Pauli matrices acting on the upper and lower 4-components of the 8-component spinor Ψ [2]. Γ_μ^{5a} satisfies the chiral Ward-Takahashi identity

$$q^\mu\Gamma_\mu^{5a}(p+\frac{q}{2},p-\frac{q}{2}) = -\frac{i}{4}\left(\Sigma\gamma^5\tau^a O_3 + \gamma^5\tau^a O_3\Sigma\right), \tag{2.5}$$

where $\Sigma = \sigma - \gamma\cdot\phi^c\gamma^5\mathcal{A}_c O_{(+)} + \gamma\cdot\bar\phi_c\gamma^5\mathcal{A}^c O_{(-)}$ is the self-energy term (without the contribution to the wave function renormalization) for the quarks and $\mathcal{A}^c_{ab} = -\mathcal{A}_{c,ab} = -\epsilon^{abc}$. ϵ^{abc} is the total antisymmetric Levi-Civita tensor in the color space of the quark. In both the Nambu phase where $\sigma\neq 0$ and $\phi^c_\mu = \bar\phi_{c\mu} = 0$ and the superconducting phase where $\phi^c_\mu \neq 0$, $\bar\phi_{c\mu}\neq 0$ and possibly $\sigma\neq 0$, Eq. 2.5 implies that Γ_μ^{5a} contains a massless Goldstone boson pole due to the fact that its right hand side (r.h.s.) is finite in the $q^2\to 0$ limit. The appearance of this massless pole in the physical excitation spectrum following the spontaneous chiral symmetry breaking is required by the Goldstone theorem.

The chiral Ward-Takahashi identity Eq. 2.5 can determine various properties of the chiral Goldstone boson. We shall consider the case in which $\phi^2 \equiv \bar\phi_{c\mu}\phi^{c\mu}\neq 0$, $\mu^\alpha = 0$ and $\sigma = 0$ that has not been discussed in the literature to demonstrate some of the elementary features of the superconducting phase related to the chiral symmetry. Eq. 2.5 becomes

$$q^\mu\Gamma_\mu^{5a}(p+\frac{q}{2},p-\frac{q}{2}) = -\frac{i}{2}\left(\gamma\cdot\bar\phi_c\mathcal{A}^c O_{(-)} + \gamma\cdot\phi^c\mathcal{A}_c O_{(+)}\right)\tau^a \tag{2.6}$$

in such a case. The propagator of the Goldstone diquark is defined as $G_\delta(q) = -it^{\mu\nu}/\Delta(q)$. The denominator $\Delta(q)$ can be generally parameterized as

$$\Delta(q) = q^2 + a_\delta\frac{(\phi\cdot q)^2}{\phi^2} \tag{2.7}$$

if we choose the phases of ϕ^c_μ and $\bar\phi_{c\mu}$ such that $\phi^c_\mu = -\bar\phi_{c\mu}$. With the following ansatz, namely,

$$t_c^{c'\mu\nu}(q\to 0) = -\frac{\phi^{c'\mu}\bar\phi_c^\nu}{\phi^2}, \bar t_{c'}^{\mu c\nu}(q\to 0) = -\frac{\bar\phi^\mu{}_{c'}\phi^{c\nu}}{\phi^2}, \tag{2.8}$$

and with the definition of the Goldstone diquark-quark vertices given by

$$\bar D_\mu^{ca}(p+\frac{q}{2},p-\frac{q}{2}) = -\frac{i}{2}g_{\delta q}\gamma_\mu\tau^a\mathcal{A}^c O_{(-)}, D^a_{c\mu}(p+\frac{q}{2},p-\frac{q}{2}) = \frac{i}{2}g_{\delta q}\gamma_\mu\tau^a\mathcal{A}_c O_{(+)}, \tag{2.9}$$

we can obtain the value of a_δ, the Goldstone diquark-quark coupling constant $g_{\delta q}$ and the Goldstone diquark decay constant f_δ by separating out the massless pole in Γ_μ^{5a}, which can be approximately evaluated by using an one loop perturbation calculation [5]. The result is

$$\begin{aligned}\left(\Gamma_\mu^{5a}\right)_{pole} = -2D^b_a\left(p+\frac{q}{2},p-\frac{q}{2}\right)\frac{-it^{\alpha\beta}}{\Delta(q)}Tr\int\frac{d^4k}{(2\pi)^4}\bar D^b_\beta\left(k-\frac{q}{2},k+\frac{q}{2}\right)\\ \frac{i}{\gamma\cdot(k+\frac{q}{2})-\Sigma}\frac{i}{2}\gamma_\mu\gamma^5\frac{\tau^a}{2}O_3\frac{i}{\gamma\cdot(k-\frac{q}{2})-\Sigma} - H.c.,\end{aligned} \tag{2.10}$$

where $H.c.$ stands for the hermitian conjugation, Tr stands for the trace operation in the Dirac, flavor, color and upper and lower 4-component spaces of Ψ and the color indices are suppressed. It can be noticed that the symmetry factor [5] for the Feynman diagram is different from the 4-component theory for fermions. After a lengthy process of evaluating the trace and performing the 4-momentum integration, the above equation together with Eq. 2.6 determine the values of a_δ, $g_{\delta q}$ and f_δ [5] as functions of ϕ^2. The numerical values for them are shown in Figs. 1-3. A Goldberger-Treiman relation for single quarks in the superconducting phase exist; it can be expressed as $g_{\delta q}f_\delta = \sqrt{\phi^2}$, which also defines the scale of f_δ.

The existence of the chiral Goldstone diquark in the superconducting phase is thus established.

III. SMALL CURRENT QUARK MASS PERTURBATION

A finite current quark mass that explicitly breaks the chiral $SU(2)_L \times SU(2)_R$ symmetry has non-trivial physical consequences. For simplicity, we shall assume that both the up and down current quarks have an identical mass m_0. Some of the consequences of a finite mass for light quarks can be studied base on the Ward-Takahashi identity given by Eq. 2.5 with the term corresponding to the divergence of the axial vector current operator taken into account, namely,

$$q^\mu \Gamma^{5a}_\mu \left(p + \frac{q}{2}, p - \frac{q}{2}\right) = \frac{i}{2} m_0 D_\pi \gamma^5 \tau^a O_3 - \frac{i}{4}\left(\Sigma\gamma^5\tau^a O_3 + \gamma^5\tau^a O_3 \Sigma\right), \tag{3.1}$$

and

$$\frac{i}{4} D_\pi \gamma^5 \tau^a O_3 = \int d^4x_1 d^4x_2 e^{ix_1\cdot(p+q/2) - ix_2\cdot(p-q/2)} \langle 0\,| T\Psi(x_1)\bar{\Psi}(x_2) j^{5a}(0)|\,0\rangle|_{amp}. \tag{3.2}$$

Here "T" stands for the time ordering, $j^{5a} = \frac{i}{4}\bar{\Psi}\gamma^5\tau^a O_3 \Psi$ and the subscript "amp" denotes the amputation of external fermion lines. D_π is a scalar function.

In the Nambu phase, if the assumption that $D_\pi(q^2 = 0)$ is dominated by the pion pole, the mass of the pion moves to a finite value, provided that to the first order in m_0, the Goldberger-Treiman relation $g_{\pi q} f_\pi = \sigma$ and the Gell-Mann, Oakes and Renner [6] (GOR) relation $f_\pi^2 m_\pi^2 = -\frac{1}{2} m_0 \langle 0\,|\bar{\Psi}\Psi|\,0\rangle$ hold. This can be checked by evaluating the r.h.s. of Eq. 3.1. The Σ term on the r.h.s. of Eq. 3.1 has a simple diagonal form

$$\Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \tag{3.3}$$

in the Nambu phase. It can be shown that Eq. 3.1 is [7]

$$q^\mu \Gamma^{5a}_\mu \left(p + \frac{q}{2}, p - \frac{q}{2}\right) = \frac{i}{2}\left(\frac{m_0}{2}\frac{g_{\pi q}}{f_\pi \sigma} G_\pi(q^2)\langle 0\,|\bar{\Psi}\Psi|\,0\rangle - 1\right)\gamma^5 \tau^a O_3, \tag{3.4}$$

where

$$G_\pi(q^2) = \frac{1}{q^2 - m_\pi^2} + \bar{R}(q^2). \tag{3.5}$$

$\bar{R}(q^2)$ is a smooth function of q^2 at small q^2 (namely, $q^2 \le m_\pi^2$). If the above mentioned Goldberger-Treiman relation and the GOR relation hold and $\bar{R}(q^2 = 0) = 0$, the r.h.s. of Eq. 3.4 vanishes at $q^2 = 0$. Therefore Γ^{5a}_μ is regular at $q^2 = 0$. It implies the disappearance of the massless pole in Γ^{5a}_μ as well as in the physical spectra.

In the superconducting phase where $\phi^2 \ne 0$ and possibly $\sigma \ne 0$, the situation is more complicated. In this case, Σ term on the r.h.s. of Eq. 3.1 takes the following form

$$\Sigma = \begin{pmatrix} \sigma & -\gamma\cdot\phi^c\gamma^5\mathcal{A}_c \\ \gamma\cdot\bar{\phi}_c\gamma^5\mathcal{A}^c & \sigma \end{pmatrix}. \tag{3.6}$$

A finite m_0 for the current quarks in this case does not render the r.h.s. of Eq. 3.1 vanish when $q^2 \to 0$. There always remains a finite strength of the massless excitation in Γ^{5a}_μ as long as the mass term is of the form $\frac{1}{2} m_0 \bar{\Psi}\Psi$. As a consequence, there are massless excitations in the superconducting phase even if the chiral $SU(2)_L \times SU(2)_R$ symmetry is explicitly broken by m_0. There is no GOR type of relation for the Goldstone diquark in the superconducting phase. In addition, certain mixing between two sets of the auxiliary fields π^a and $(\delta^{ca}_\mu, \bar{\delta}^a_{c\mu})$ [2] is needed to represent the Goldstone boson excitation in the superconducting phase even when $m_0 \ne 0$ and $\sigma = 0$. The above mentioned mixing provides us with one of the motivations for the following extension of the PCAC relation.

IV. THE EXTENSION OF THE PCAC RELATION AND CURRENT ALGEBRA RESULTS

The present day large scale strong interaction vacuum is expected to be in the Nambu phase. There are a few obvious reasons for this statement. First, the overwhelming color confinement at the present day condition prevents the large scale superconducting phase in the strong interaction vacuum scenario from been acceptable. Second, the long range strong interaction force in the superconducting phase due to the massless Goldstone diquark excitation

inside the hadronic system is absent in the experimental observations. However, localized superconducting phases inside a baryonic system are not implausible possibilities.

Due to the above considerations and the ones given in the introduction, we explore in this section a subset of the phenomenological consequences of the possibility in which the interior of a nucleon contains diquark condensation that spontaneously breaks the chiral $SU(2)_L \times SU(2)_R$ symmetry down to an flavor (isospin) $SU(2)_V$ symmetry. The study of Ref. [8] indicates that the model Lagrangian introduced in Ref. [2] indeed support such a scenario when the coupling constant α_3 is sufficiently large.

The on shell matrix elements of the axial vector current operator between single nucleon states can be parameterized as

$$\langle p' | A^a_\mu(0) | p\rangle = \bar{U}(p') \left(g_A \gamma_\mu + g_P q_\mu + g_T \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} \right) \gamma^5 \frac{\tau^a}{2} U(p), \tag{4.1}$$

with $q_\mu = (p' - p)_\mu$, m_N the mass of a nucleon and U(p) the 4-component nucleon spinor. The longitudinal piece $g_P q_\mu$ on the r.h.s. of Eq. 4.1 is dominated by the contributions of the Goldstone bosons of the spontaneous chiral symmetry breaking. If only the Nambu phase is considered, g_P is given by

$$g_P(q^2) = -2 \frac{g_{\pi NN}(q^2) f_\pi(q^2)}{q^2 - m_\pi^2} + \bar{g}_P(q^2), \tag{4.2}$$

with $\bar{g}_P(q^2)$ the residue term and $g_{\pi NN}(q^2) f_\pi(q^2)$ a slow varying function of both m_π^2 and q^2. If, however, the assumption that there is a diquark condensation inside a nucleon is made, there world be another longitudinal term in the matrix elements of the axial vector current operator due to the Goldstone diquark excitation inside that nucleon. The expression for g_P has to be modified to

$$g_P(q^2) = -2 \left(\frac{g_{\pi NN}(q^2) f_\pi(q^2)}{q^2 - m_\pi^2} + z_\delta g_{\delta N}(q^2) f_\delta(q^2) \eta(q^2) \right) + \bar{g}_P(q^2) \tag{4.3}$$

after considering this additional excitation. Here $\eta(q^2)$ is related to the propagator of the Goldstone diquark excitation and z_δ is a constant. Similar to the pion, we introduce $g_{\delta N}$ as the Goldstone diquark-nucleon coupling constant and f_δ as the Goldstone diquark decay constant. Albeit there is massless excitation in Γ^{5a}_μ, there is no pole behavior in $\eta(q^2)$ in the small q^2 region due to the fact that a Goldstone diquark carries color so that it, like a quark, is confined inside the nucleon.

At the operator level, the divergence of the axial vector current operator is

$$\partial^\mu A^a_\mu = \frac{1}{2} m_0 \bar{\Psi} i \gamma^5 \tau^a O_3 \Psi. \tag{4.4}$$

The PCAC relation is given by

$$\partial^\mu A^a_\mu = -f_\pi m_\pi^2 \phi^a_\pi, \tag{4.5}$$

which makes an implicit assumption that the quark bilinear operator $\bar{\Psi} i \gamma^5 \tau^a O_3 \Psi$ couples only to the pion excitation in the low momentum transfer regime. It can be regarded as a definition of the pion field (when going off the pion mass shell). Taking the matrix elements of Eq. 4.5, it can be shown that this definition is inconsistent with Eq. 4.3 due to the additional term added to g_P. We have at least two choices. The first one is to reject Eq. 4.3, which we shall not do in this paper. The second one is to modify Eq. 4.5 when its matrix elements are taken. We take the following assumption that

> **Assumption I:** *The divergence of the axial vector current operator is dominated by the longitudinal chiral Goldstone bosons, namely, the pion and the possible Goldstone diquark, contributions in the low momentum transfer region.*

in the sequel. The content of it, especially the range of q^2 in which it is valid, will be explained in the next section in more detail

For a nucleon, it can be specified as

$$\langle p' | \partial^\mu A^a_\mu | p\rangle = -\langle p' | \left(f_\pi m_\pi^2 \phi^a_\pi + f_\delta s_\delta \phi^a_\delta \right) | p\rangle, \tag{4.6}$$

with $s_\delta \sim m_\pi^2$ a parameter proportional to m_0 that characterizes the strength of mixing of the Goldstone diquark excitation within the $\bar{\Psi} i \gamma^5 \tau^a O_3 \Psi$ operator in the presence of baryonic matter and ϕ^a_δ a pseudo-scalar [9] driving field

for the Goldstone diquark excitation inside the nucleon. The physical meaning of Eq. 4.6 is that in a baryonic system, the operator $\partial^\mu A^a_\mu = \frac{1}{2} m_0 \bar\Psi i\gamma^5 \tau^a O_3 \Psi$ can excite two sets of distinct longitudinal chiral soft modes (Goldstone bosons in the chiral symmetric limit) if there is diquark condensation.

From Eqs. 4.1 and 4.6, one obtains the following equation

$$\begin{aligned} 0 = & q^2 \left[2m_N g_A(q^2) + (q^2 - m_\pi^2) g_P(q^2)\right] \\ & +2m_\pi^2 \left[g_{\pi NN}(q^2) f_\pi(q^2) + (q^2 - m_\pi^2)\frac{s_\delta}{m_\pi^2} g_{\delta N}(q^2) f_\delta(q^2)\eta(q^2) - m_N g_A(q^2)\right]. \end{aligned} \tag{4.7}$$

If the assumption that

Assumption II: $2m_N g_A(q^2) + (q^2 - m_\pi^2) g_P(q^2)$ *is a slow varying function of* q^2 *and* m_π^2

is made, two equations follow, namely,

$$g_P(q^2) = -2\frac{m_N g_A(q^2)}{q^2 - m_\pi^2}, \tag{4.8}$$

$$m_N g_A(q^2) = g_{\pi NN}(q^2) f_\pi(q^2) + (q^2 - m_\pi^2)\frac{s_\delta}{m_\pi^2} g_{\delta N}(q^2) f_\delta(q^2)\eta(q^2). \tag{4.9}$$

The slow varying assumption of these quantities together with the modified PCAC relation, Eq. 4.6, imply that the residue term $\bar g_P(q^2)$ in Eq. 4.3 is unimportant in the small q^2 regime, which is consistent with the assumption I. Eqs. 4.8, 4.9 are consistent with Eq. 4.3 provide that $z_\delta = s_\delta/m_\pi^2$. The modified Goldberger-Treiman relation is obtained when $q^2 = m_\pi^2$ is assumed on the r.h.s. of the second equation of Eq. 4.9 and $q^2 \to 0$ is taken on its left hand side (l.h.s.), namely,

$$m_N g_A(0) = g_{\pi NN}(m_\pi^2) f_\pi(m_\pi^2) + \lim_{q^2\to m_\pi^2} (q^2 - m_\pi^2) z_\delta g_{\delta N}(q^2) f_\delta(q^2)\eta(q^2). \tag{4.10}$$

Assuming the results of Ref. [10] can be used in the time like region for q_μ, the extrapolation of g_A from $q^2 = m_\pi^2$ to $q^2 = 0$ introduces an error of order $m_\pi^2/M_A^2 \sim 1\%$ with $M_A \sim 1GeV$ [10]. Since there is no pole in q^2 for $\eta(q^2)$ at m_π^2, Eq. 4.10 reduces to

$$m_N g_A(0) = g_{\pi NN}(m_\pi^2) f_\pi(m_\pi^2), \tag{4.11}$$

namely, the Goldberger-Treiman relation in a different form from the one presented in the literature, which is

$$m_N g_A(0) = g_{\pi NN}(0) f_\pi(0). \tag{4.12}$$

Another form of the Goldberger-Treiman relation, which is called the on shell Goldberger-Treiman relation, is important for the discussions to be presented. It is

$$m_N g_A(m_\pi^2) = g_{\pi NN}(m_\pi^2) f_\pi(m_\pi^2). \tag{4.13}$$

Eqs. 4.8 and 4.9 are the main results following assumptions I and II.

The modification of some of the current algebra results related to PCAC due to a possible Goldstone diquark excitation inside a nucleon can also be investigated. The Adler-Weisberger sum rule and the nucleon Σ_N term will be studied in this paper base on a Ward identity involving the axial vector current operator [11–13]. In order to obtain useful information, the low lying longitudinal excitation contributions and the rest part of the axial vector current operator inside a time ordered product are separated in the following way

$$\begin{aligned} \left\langle T\left(\ldots A^a_\mu \ldots\right)\right\rangle = & \left\langle T\left(\ldots \bar A^a_\mu \ldots\right)\right\rangle + \\ & \partial_\mu \left\langle T\left(\ldots f_\pi \phi^a_\pi \ldots\right)\right\rangle + \partial_\mu \left\langle T\left(\ldots z_\delta f_\delta \phi^a_\delta \ldots\right)\right\rangle, \end{aligned} \tag{4.14}$$

with the second and the third terms on the r.h.s. the longitudinal parts of A^a_μ, which is dominated by the low lying chiral Goldstone boson contributions, and $\bar A^a_\mu$, which is expected to change slowly with the momentum transfer q^μ when its matrix elements are taken between nucleon states, containing the rest part of A^a_μ. The matrix elements are evaluated on the pion mass shell [13]. As a rule, which is going to be explored in more detail in other work [14], those of $\bar A^a_\mu$ that connect nucleon state to other hadronic intermediate states by a gradient-coupling [15] are then extrapolated to the kinematic point where $q^2 = 0$ in order to compare with experimental data by taking the assumption that

Assumption III: *they are slow varying function of q^2.*

The error of the extrapolation is expected to be of order $O(m_\pi^2/M_A^2) \sim 1\%$. There are contributions from other off (nucleon) shell terms and baryonic excitations in the intermediate states if $q^2 \neq 0$; At finite q^2, the contribution from the single nucleon pole can not be separated from the background. The extrapolation to $q^2 = 0$ allows the extraction of single nucleon axial vector form factor $g_A(0)$. The modified Adler-Weisberger sum rule can be shown to have the following form

$$\begin{aligned} g_A^2(q^2=0) = 1 - 2\frac{f_\pi^2(m_\pi^2)}{\pi}\int_{m_\pi}^{\infty} d\nu \frac{\sigma_{tot}^{\pi^- p}(\nu) - \sigma_{tot}^{\pi^+ p}(\nu)}{(\nu^2 - m_\pi^2)^{1/2}} \\ -2\lim_{q^2\to m_\pi^2}(q^2-m_\pi^2)^2 f_\delta^2(q^2)\lim_{\nu\to 0}\frac{z_\delta^2}{\nu}G_{\delta N}^{(-)}(\nu,0,q^2,q^2), \end{aligned} \tag{4.15}$$

where $G_{\delta N}^{(-)}$ is related to the flavor odd forward Goldstone diquark-nucleon scattering amplitude, which is driven by ϕ_δ^a, without the amputation of the diquark lines. Assumption III is nontrivial. It is, however, supported by observations. This is discussed in section VI.

The expression for the nucleon Σ_N term, which is of order m_0, is modified to

$$\Sigma_N = f_\pi^2(m_\pi^2)T_{\pi NN}^{(+)}(0,0,m_\pi^2,m_\pi^2) + \lim_{q^2\to m_\pi^2}(q^2-m_\pi^2)^2 f_\delta^2(q^2)z_\delta^2 G_{\delta N}^{(+)}(0,0,q^2,q^2), \tag{4.16}$$

with $G_{\delta N}^{(+)}$ related to the flavor even forward Goldstone diquark-nucleon scattering amplitude without the amputation of the diquark lines. The second term on the r.h.s. of Eq. 4.16 vanishes on the pion mass shell.

V. THE CONTENT OF ASSUMPTIONS I AND II AND THEORETICAL UNCERTAINTIES

Before confront the results of the previous sections with observations, we assess the possible theoretical uncertainties involving assumptions I and II to ascertain the degree of confidence that one should assume for the discussions in the following sections.

Assumptions I and II are sufficient conditions for Eqs. 4.8 and 4.9. They are, however, not necessary ones for Eq. 4.8. This is because Eq. 4.8 represents the conservation of axial vector current in the chiral limit of $m_0 \to 0$ or in the high momentum transfer limit of $q^2 >> m_\pi^2$. The true non-trivial q^2 region for Eq. 4.8 consists of low q^2, where the axial vector current is not conserved. The q^2 region in which Eq. 4.9 hold contains the q^2 region where assumptions I and II are true. It can be parameterize as $-\zeta_- s_{th} < q^2 < \zeta_+ s_{th}$ with $s_{th} \sim 1$ GeV^2 and the values for ζ_- and ζ_+ determined in the following. The above statements can be justified through a more detailed analysis of the content of assumption II by relaxing the constraint imposed by assumption I. To this end, Eq. 4.7 is rewritten as

$$q^2 A(q^2,m_\pi^2) + m_\pi^2 B(q^2,m_\pi^2) = m_\pi^2 C(q^2,m_\pi^2), \tag{5.1}$$

with

$$A(q^2,m_\pi^2) = m_N g_A(q^2) + \frac{1}{2}(q^2-m_\pi^2)g_P(q^2), \tag{5.2}$$

$$B(q^2,m_\pi^2) = g_{\pi NN}(q^2)f_\pi(q^2) + (q^2-m_\pi^2)\frac{s_\delta}{m_\pi^2}g_{\delta N}(q^2)f_\delta(q^2)\eta(q^2) - m_N g_A(q^2), \tag{5.3}$$

and $C(q^2,m_\pi^2)$ to be defined in the following. $C(q^2,m_\pi^2) = 0$ when assumption I is valid. Here the m_π^2 dependence of A, B and C is written out explicitly. The following equations hold, namely,

$$\lim_{q^2\to\infty} A(q^2,m_\pi^2) = 0, \tag{5.4}$$

$$\lim_{m_0\to 0} A(q^2,m_\pi^2) = 0, \tag{5.5}$$

which represents the conservation of axial vector current in the $q^2 >> m_\pi^2$ region and in the chiral symmetric limit ($m_0 \to 0$). Assumption II is non-trivial since it implies $A(q^2,m_\pi^2) \approx \lim_{m_0\to 0} A(q^2,m_\pi^2) = 0$, which establishes Eq. 4.8 for any q^2.

If $A(q^2,m_\pi^2) = 0$, then Eq. 5.1 reduces to

$$B(q^2, m_\pi^2) = C(q^2, m_\pi^2). \tag{5.6}$$

That $C(q^2, m_\pi^2)$ is small is the true content of assumption I. This can be seen from the following analysis.

Let's define an error operator for our extended PCAC relation,

$$m_\pi^2 \hat{\Delta}^a(x) = \partial^\mu A_\mu^a(x) - m_0 \chi^a(x), \tag{5.7}$$

with the longitudinal chiral soft excitation modes driving field $\chi^a(x)$

$$\chi^a(x) = \frac{1}{2} P(s_{th}) \bar{\Psi}(x) i\gamma^5 \tau^a O_3 \Psi(x) P(s_{th}), \tag{5.8}$$

where the projection operator $P(s_{th})$ is defined as

$$P(s_{th}) |\, s\rangle = 0 \tag{5.9}$$

for any state with pionic quantum number and invariant mass $s > s_{th}$ and

$$P(s_{th}) |\, s\rangle = |\, s\rangle \tag{5.10}$$

for any state with pionic quantum number and invariant mass $s \leq s_{th}$. The value for s_{th} is determined in the following. The matrix element of $\hat{\Delta}^a(0)$ between single nucleon states is defined as

$$C(q^2) \bar{U}(p') i\gamma^5 \tau^a U(p) = \langle\, p' | \hat{\Delta}^a(0) |\, p\rangle, \tag{5.11}$$

where $C(q^2)$ is identical to the $C(q^2, m_\pi^2)$ in Eq. 5.1. An once subtracted dispersion relation for $C(q^2)$ can be used, namely,

$$C(q^2) = C(m_\pi^2) + \frac{q^2 - m_\pi^2}{\pi} \int_{s_{th}}^\infty ds' \frac{ImC(s')}{(s' - q^2)(s' - m_\pi^2)}. \tag{5.12}$$

At low q^2, the value of the integral on the r.h.s. of the above equation is insensitive to the shape of $ImC(s)$ as a function of s, so it can be replaced by a step function of the following form

$$ImC(s) = \alpha \theta(s - s_{th}) \tag{5.13}$$

for our purpose, with

$$\alpha \sim C(m_\pi^2). \tag{5.14}$$

The above relation is a consequence of the property that in an asymptotic free theory like QCD, the form factor $C(q^2)$ actually satisfies an unsubtracted dispersion relation due to the fact that $ImC(q^2)$ falls off faster than a constant at large momentum transfer squared s, where the dominant contribution to the matrix element of operator $\hat{\Delta}(0)$ between initial vacuum state and hadronic final states is through its coupling to the current quark-antiquark lines. The once subtracted form of dispersion relation is used to emphasize the small value of $C(m_\pi^2)$ in observations.

This allows the integration in Eq. 5.12 to be done explicitly

$$C(q^2) = C(m_\pi^2) + \frac{\alpha}{\pi} ln \left(\frac{s_{th} - q^2}{s_{th} - m_\pi^2} \right). \tag{5.15}$$

In general $C(q^2)$ is not necessarily small at low q^2 since the term corresponding to it is of order m_π^2 already. Theoretically, the smallness of this quantity at low q^2 is a non-trivial result which can be argued by using the asymptotic freedom property of QCD. The fact $C(m_\pi^2)$ is small can be deduced from the success of the Goldberger-Treiman relation on the pion mass shell (without extrapolation in q^2) given by Eq. 4.13 in the empirical observations [16]. The error for it has an order of magnitude of around 1%. The range of q^2 in which assumption I is valid can be obtained by requiring $\delta C(q^2) \equiv |C(q^2) - C(m_\pi^2)|$ to be less than or equal to $|C(m_\pi^2)|$. This gives $q^2 < 0.7 s_{th}$. The range of validity of the above arguements in the negative q^2 region is much larger than 1 GeV^2 in magnitude.

The next step is to estimate the value of s_{th}. The lowest value of s_{th} in the pionic channel is below $9m_\pi^2$, which corresponds to an anomalous threshold for the three pion state. However, the effective s_{th} correspond to that of the $\rho\pi$ threshold, which is considerablly larger than $9m_\pi^2$. This is a consequence of the fact that the underlying dynamics of QCD is chiral invariant except for a small mass term. From this fact the dynamical (operator) equation

Eq. 4.4 follows. This dynamical equation ensures that the operator $\bar{\Psi} i\gamma^5\tau^a O_3\Psi$ can only excite a longitudinal vector excitation since it is proportional to the divergence of a axial vector operator field. Therefore the state in which the three pions are all in a s-state is dynamically forbiden. The allowed state which dominates the dispersion relation is the one where two of the three pions have a relative angular momentum of 1, which is itself dominated by the ρ excitation strength. Therefore $s_{th} \sim (m_\rho + m_\pi)^2$ and the range of q^2 in which assumption I and II is valid is 30 to 35 times larger than $m_\pi^2 \approx 0.02 GeV^2$.

In conclusion, both of assumptions I and II are supported by observations and the physical picture based on the smallness of the current quark masses. Assumption I is connected to the success of the on pion mass shell (without q^2 extrapolation) Goldberger-Treiman relation given by 4.13; assumption II is connected to the verification of Eq. 4.8 in an experimental investigation [17].

VI. EMPIRICAL BASIS

A. Goldberger-Treiman relation

The Goldberger-Treiman relation is satisfied to about 5% in experimental observations. This good agreement indicates the validity of the various assumptions combined made above and a rather small value of $\lim_{q^2\to m_\pi^2}(q^2 - m_\pi^2) z_\delta g_{\delta N}(q^2) f_\delta(q^2)\eta(q^2)$. The $\eta(q^2)$ term on the pion mass shell actually vanishes since $\eta(q^2)$ does not has a pole in the low q^2 region. However, a small value for the $\eta(q^2)$ term does not follow from the success of the Goldberger-Treiman relation.

Eq. 4.12 is the other form of the Goldberger-Treiman relation used in the literature. It can be made to contain an error of less than 1% if soft $g_{\pi NN}(q^2)$ is used [16]. Eq. 4.13 is an equation that is satisfied in observation to an accuracy of 1%. The accuracy of Eq. 4.13 in observations forms one of the empirical basis for assumption I. This point is discussed in some detail in section V.

B. Nucleon axial vector form factor g_A and pseudoscalar form factor g_P

A recent experimental study [17] provides a strong support [18] of Eq. 4.8 within momentum transfer region of $0 < -q^2 < 0.2\ GeV^2$.

C. g_A and the pion-nucleon coupling constant $g_{\pi NN}$

Eq. 4.9 gives a specific relation between $g_A(q^2)$ and $g_{\pi NN}(q^2)f_\pi(q^2)$ if no other chiral soft modes bellow $q^2 = (m_\rho + m_\pi)^2$ is present. We study whether or not Eq. 4.9 without the Goldstone diquark contribution term, namely,

$$m_N g_A(q^2) = g_{\pi NN}(q^2) f_\pi(q^2) \tag{6.1}$$

is consistent with phenomenology.

The q^2 dependence of $g_A(q^2)$ in the space-like q^2 region is of a dipole form [10], namely,

$$g_A(q^2) = \frac{g_A(0)}{(1 - q^2/M_A^2)^2}, \tag{6.2}$$

with $M_A \approx 1$ GeV. We shall use $M_A = 1.0$ GeV in the following.

The q^2 dependence of $g_{\pi NN}(q^2)$ is known less well than that of $g_A(q^2)$. A monopole form for it, which can be parameterized as

$$g_{\pi NN}(q^2) = g_{\pi NN}\frac{\Lambda_{\pi NN}^2 - m_\pi^2}{\Lambda_{\pi NN}^2 - q^2} \tag{6.3}$$

is agreed upon in the literature; the value for $\Lambda_{\pi NN}$ varies. It is found to be greater than 1.2 GeV in nucleon-nucleon (NN) scattering and deuteron property studies [19]. This range of $\Lambda_{\pi NN}$ implies a rather small quark core for a nucleon. It is in contradiction with the expectations of many chiral nucleon models for the nucleon (see, e.g. Refs. [20]– [24]). Lattice QCD evaluation [25] also indicates a smaller one, namely, $\Lambda_{\pi NN} \sim 800$ MeV. On the phenomenological side, Goldberger-Treiman discrepancy study [16], $pp\pi^0$ v.s. $pn\pi^+$ coupling constant difference [26],

high energy pp scattering [27] and charge exchange reaction [28], etc, support a value of $\Lambda_{\pi NN}$ close to 800 MeV. By introducing a second "pion" π' with mass 1.3 GeV, $\Lambda_{\pi NN}$ can be chosen to be around 800 MeV without spoiling the fit to the NN scattering phase shifts and deuteron properties [29]. This picture was later justified by a microscopic computation in Ref. [30].

The q^2 dependence of $f_\pi(q^2)$ is little known from experimental observations. It can be expressed in terms of a once subtracted dispersion relation, namely,

$$f_\pi(q^2) = f_\pi(m_\pi^2) + \frac{q^2 - m_\pi^2}{\pi} \int_{s_{th}}^{\infty} ds' \frac{Imf_\pi(s')}{(s'-q^2)(s'-m_\pi^2)}. \tag{6.4}$$

Since the lightest physical state connects to the axial vector current operator with the quantum number of pion is the $\rho\pi$ two particle state, the value of s_{th} is chosen to be $(m_\rho + m_\pi)^2$, where $m_\rho = 770$ MeV. At $s' = 4m_N^2$, which correspond to the lowest invariant mass of a $\bar{N}N$ system, another branch cut for $f_\pi(q^2)$ develops. We shall include $\rho\pi$ state only since $\bar{N}N$ state contributions to Eq. 6.4 is small when q^2 is small. So Eq. 6.4 can be written as

$$f_\pi(q^2) = f_\pi(m_\pi^2) + \frac{q^2 - m_\pi^2}{\pi} \int_{s_{th}}^{4m_N^2} ds' \frac{Imf_\pi(s')}{(s'-q^2)(s'-m_\pi^2)}. \tag{6.5}$$

Our next step involves the specification or computation of $Imf_\pi(s)$ by exploring the fact that only the $\rho\pi$ state which couples to the pion contributes to $Imf_\pi(s)$ in the momentum transfer region of interest to this paper. An one loop computation to $Imf_\pi(s)$ is known to be insufficient to account for experimental data in other studies [29,30], the $\rho\pi$ correlation, which forms a resonance near 1.3 GeV, is required. We therefore propose the following form for $Imf_\pi(s)$,

$$Imf_\pi(q^2) = \frac{3}{4} \frac{m_N}{g_{\pi NN}} \frac{g_{\rho\pi\pi}^2}{4\pi} \bar{\rho}_{\pi,\rho\pi}(q^2) \tag{6.6}$$

with the reduced density of state

$$\bar{\rho}_{\pi,\rho\pi}(q^2) = \bar{\rho}_0(q^2) \left(1 + \frac{\lambda I_B}{(q^2 - s_B)^2 + \bar{\rho}_0^2(q^2) I_B^2}\right) \tag{6.7}$$

and

$$\bar{\rho}_0(q^2) = \sqrt{1 - \frac{(m_\rho + m_\pi)^2}{q^2}} \sqrt{1 - \frac{(m_\rho - m_\pi)^2}{q2}} \left(1 - \frac{m_\rho^2 - m_\pi^2}{3q^2}\right) \theta[q^2 - (m_\rho + m_\pi)^2], \tag{6.8}$$

where $\theta(x)$ is the step function with a value of unity for positive x, s_R is chosen to be 1.69 GeV^2, λ characterizes the strength of the $\rho\pi$ resonance in $Imf_\pi(q^2)$ and $\bar{\rho}_0(s_R)I_B$ characterizes the width of the resonance.

The above form is chosen so that when $\lambda = 0$, $Imf_\pi(q^2)$ is the one loop result in the Feynman-t' Hooft gauge (for the ρ propagator). The $\rho\pi\pi$ interaction piece of Lagrangian density used for evaluation of $Imf_\pi(q^2)$ is

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \epsilon^{abc} \pi^a \partial^\mu \pi^b \rho_\mu^c. \tag{6.9}$$

The corresponding piece of the axial vector current operator, which can be obtained from the Noether theorem, can be written as

$$A_\mu^a = g_{\rho\pi\pi} \frac{m_N}{g_{\pi NN}} \epsilon^{abc} \pi^b \rho_\mu^c, \tag{6.10}$$

where use has been made of the linear σ model [31] relation $m_N = g_{\pi NN}\sigma$.

The value for λ, $\Lambda_{\pi NN}$ and I_B is adjusted so that the minimum value of the following function

$$f(\lambda, \Lambda_{\pi NN}, I_B) = \frac{1}{N} \sum_{k=0}^{N} \frac{\left[m_N g_A(q_k^2) - g_{\pi NN}(q_k^2) f_\pi(q_k^2)\right]^2}{m_N^2 g_A^2(q_k^2)} e^{3q_k^2}, \tag{6.11}$$

with

$$q_k^2 = q_{min}^2 + k \frac{q_{max}^2 - q_{min}^2}{N}, \tag{6.12}$$

is achieved. The value of N is chosen to be 100. $q^2_{min} = -0.6$ GeV and $q^2_{max} = 0.2$ GeV. The factor e^{3q^2} is used to put more weight on small $|q^2|$ region where the fit tends to be poor.

In all cases listed in Table I, $\Lambda_{\pi NN} < 0.95$ GeV. If a value $g^2_{\rho\pi\pi}/4\pi = 1.0$ is taken, the qualitative shape of $Imf_\pi(q^2)$ in Fig. 4 obtained by a minimization of Eq. 6.11 is similar to the $Im\Gamma(q^2)$ of Ref. [30]. Quantitatively, it has a broader width. The phenomenological value for $g_{\rho\pi\pi}$ can be deduced from the $\rho \to \pi\pi$ decay process. It has a value satisfies $g^2_{\rho\pi\pi}/4\pi \approx 2.9$. Using this value of $g_{\rho\pi\pi}$, $Imf_\pi(q^2)$ is obtained by minimization of Eq. 6.11. The result is given in Table I and plotted in Fig. 4. It's drastically different in shape from that of $Im\Gamma(q^2)$ in Ref. [30]. In fact, instead of increasing the density of states relative to the one loop result (Eq. 6.6 with $\lambda = 0$), the resonance contribution decreases the density of states in order to satisfy Eq. 6.1. The reduction of density of state indicates either the solution is unphysical (for a normal resonance) or there is a competing resonance in another channel that couples to the pionic channel we are dealing with. But what can the "other resonance" channel be? There is no known resonance there. Therefore, the result Eq. 6.1 obtained from chiral symmetry argument without diquark condensation for a nucleon is inconsistent with phenomenology. The introduction of the Goldstone diquark contribution term seems to be inevitable.

D. Adler-Weisberger sum rule

The value of g_A from nuclear β decay experiments is 1.261 ± 0.004 [39]. The value of the same quantity obtained from the Adler-Weisberger sum rule using pion nucleon scattering cross section (and after estimating other corrections) is about 1.24 ± 0.02 [11]. These two numbers, with a central g_A value deviation of order 1.3%, provide a support for assumption III. Again, the contribution of the second term on the r.h.s. of Eq. 4.15 is expected to be zero on the pion mass shell. Therefore the good agreement of the Adler-Weisberger sum rule with the observation does not necessarily constitute a fact that is against the additional terms added in Eqs. 4.6 and 4.14.

VII. OTHER EMPIRICAL OBSERVATIONS

Several empirical observations, which are considered relevant to this paper and for which the present understanding for them in terms of conventional picture is considered to have difficulties are listed and discussed in this section. Since the previous theoretical understanding of the experimental observations had not taken into account of the possibility of diquark condensation inside a nucleon/nucleus, and, in addition, some of the failure of the conventional explanation for them is not unambiguous at the present level of theoretical and/or experimental precision, they are considered more as discussions than evidences. It serves the purpose of either sharpening the problems, which is expected to lead to further investigations, or showing that the introduction of diquark condensation inside a nucleon/nucleus does not contradict well established experimental facts.

A. g_P in a nucleus

The effects of the Goldstone diquark inside a nucleon, if exist, might be comparatively large in the kinematic region off the pion mass shell. Current experimental data considered do not allow any conclusive statement to be made on this point. The value of g_P can be compared to the PCAC value (Eq. 4.2) to detect the possible effects of the Goldstone diquark. Experimental determinations of g_P for a nucleon in muon capture experiments involving light nuclei show a systematic increase (of order as large as 100%) of the value of g_P from its PCAC one [32]. The world average value of g_P obtained from the muon capture experiments on a hydrogen is close to the one given by the PCAC one [33]. However, the interpretation of the results is not unambiguous (Ref. [33] and Gmitro and Truol in Ref. [32]). There are two recent measurements of the muon capture rate on the deuterium. The central value of the first measurement [34] implies a value of g_P smaller than the PCAC one [35]. The central value of the second measurement [36] implies a value of g_P close to the PCAC one [35]. The problems related to the value of g_P are not yet completely settled [37]. Polarized β decay experiments offer alternative means of measuring the value of g_P in a different range of q^2 [38]. The experiments are difficult but in principle possible.

Theoretically, the agreement of the PCAC value for g_P and the experimentally measured one is expected for a nucleon in free space. This is because Eq. 4.8 implies the deviation of g_P from the PCAC one is related to the deviation of the value of g_A, m_N and m_π from the experimentally observed ones, which is not true. Therefore, the system in which a deviation of the strength of the pionic longitudinal modes in the axial vector current operator from the PCAC one can be observed is inside a nucleus with relatively large number of nucleons. In these systems, the

coupling between the pions outside of the nucleons and the Goldstone diquark excitations inside the nucleons could render the properties of pions to change to such a degree that a deviation from PCAC value can be observed. More work is clearly needed on this subject.

B. The nucleon σ_N term

Experimental verifications of Eq. 4.16 (without the second term on its r.h.s.) have so far been unsuccessful without assuming certain strange quark content of a nucleon, which is discouraged by the OZI rule [40]. A recent review of the nucleon Σ_N problem can be found in Ref. [41]. The experimental data for the scalar form factor of the nucleon is known from experimental observations only on the Cheng-Dashen point at $t = 2m_\pi^2$. In order to make a connection between this piece of experimental information with one for the static scalar density $\sigma(0)$ that can be obtained, within some specific models, from baryonic spectra. The extrapolation of the scalar density from the Cheng-Dashen point to the $t = 0$ static point can be done using essentially model independent dispersion relation method [41]. A value of $\sigma(0) = 45$ MeV is obtained from the experimental πN scattering data.

The computations of the value of $\sigma(0)$, which relies upon the information contained in the baryonic spectra, can be classified into two categories: (1) self-consistent mean field approximation without the pion loop corrections [42] (2) chiral perturbation theory [43] with loop corrections [44].

One of the problems of the evaluation of $\sigma(0)$ from the baryonic spectra is related to the possible non-linearity in the current strange quark mass which renders the extracted value of $\sigma(0)$ from baryonic spectra unreliable. In the computations of the first category, the non-linearity is large in Nambu Jona-Lasinio type of models with contact interaction [42]; it is found to be small in models [45] where extended gluon propagators are used. A value of order 49.3 MeV is obtained for $\sigma(0)$ in Ref. [42]. It is already somewhat larger than 45 MeV obtained form the πN scattering data. The computation given by Ref. [44], which belongs to the second category, obtains corrections to the $\sigma(0)$ term due to pion loop contributions of order 10 MeV. Ref. [46] computed the $\pi\Delta$ components for a nucleon using bag and soliton models, which obtain an additional 6-10 MeV corrections.

Neither the computations given by Ref. [42] nor the one given by Ref. [44] is complete. The former ones miss the low lying collective modes and the later ones contain large corrections to σ, which imply self-consistent computations are needed. Had we, for the purpose of estimation, simply add all the above mentioned corrections up, we would have gotten a number for $\sigma(0)$ of order 68 MeV. It is much larger than the one obtained from πN scattering data. It is even larger than the un-extrapolated value of $\sigma(2m_\pi^2)$ of order 60 MeV.

The problem is still not well understood.

It should be pointed out here that an additional source of correction need to be considered to relate Σ, which is proportional to the scalar density of the nucleon on the Cheng-Dashen point [41], to $\sigma(0)$ related to the static scalar density of a nucleon. In the Hartree-Fock approximation, the static scalar density measured by σ is free of radiative corrections due to the fact that it satisfies a "gap equation", which self-consistently adjust the radiative corrections to σ to zero (cancel) by changing its value. This can be proven in the case that σ is space-time independent [5]. We expect it to be true also for $\sigma(0)$. This statement is not true if the matrix elements of the scalar density operator $\frac{1}{2}\bar\Psi\Psi$ between states of different 4-momentum are taken. Various corrections due to the interaction have to be considered.

$\Sigma = \sigma(2m_\pi^2)$ term is written as $\Sigma = \sigma_N(0) + \Delta_N$, $\sigma_N(0) = 25MeV$ can be obtained from the baryon spectra [41]. Δ_N contains various corrections due to pionic modes [41] to the extrapolation. If there is diquark condensation in a nucleon, there would be contributions to Δ_N that are proportional to ϕ^2 coming from the interaction terms. The existence of such a term can be demonstrated at the quark level by considering a second order correction to σ_N due to the interaction, which has the generic form $\delta\sigma_N^{(2)} \sim \langle p'\,|T\left(\Psi\bar\Psi\right)\left(\bar\Psi\Gamma\Psi\right)\left(\bar\Psi\bar\Gamma\Psi\right)|\,p\rangle$, with Γ and $\bar\Gamma$ a pair of the interaction vertices. This term contains ϕ^2 contributions if there is diquark condensation inside a nucleon [47]. Detailed evaluation of these terms can not be proceeded before a specific model is given. Here, we simply parameterize the effects of ϕ^2 as $\Sigma_N = \sigma_N(0) + \Delta_N^{(0)} + \beta_N\phi_N^2$, where $\Delta_N^{(0)}$ is related to the total correction that has already been considered in the literature (see, e.g., Ref. [41]). The new term depending on ϕ_N^2 is due to the possible diquark condensation inside a nucleon with an average strength measured by $\sqrt{\phi_N^2}$. It is unclear at present whether the additional term increases ($\beta_N < 0$), decreases ($\beta_N > 0$) or even eliminates the above mentioned discrepancy.

C. Deep inelastic scattering

The deep inelastic scattering (DIS) of charged leptons and neutrinos with a nucleon and a nucleus provide another way of studying the constituent quark (elementary excitation) inside that system. Of those relevant, we emphasize in particular the change in behavior of the constituent quark in a system with diquark condensation. It was discussed

in Ref. [2] that diquark condensation spontaneously breaks the $U(1)$ symmetry corresponding to the baryon number conservation leading to a superconducting phase inside the system. In this phase, the isoscalar charge of the constituent quark, albeit does not disappear, is spreaded, which can only be partially observed in the DIS. This phenomenon is known in the study of superconducting condensed matter system [48]. The advantage of considering this particular aspect of the constituent quark is related to the fact that there exist sum rules for the quark based on the localization of the isoscalar charge, which is violated in the superconducting phase. In such a case, the electric charge of some [5] of the partons, which are the up and down quarks, have only isovector charge $\pm 1/2$ in the limit of large strength of diquark condensation. Therefore if there is a diquark condensation that generates a localized superconducting phase in a nucleon, the $F_1(x)$ and $F_2(x)$ structure functions should be written as

$$F_1(x) = \frac{1}{2}\sum_i \tilde{Q}_i^2 f_i(x), \tag{7.1}$$

$$F_2(x) = \sum_i \tilde{Q}_i^2 x f_i(x), \tag{7.2}$$

where i enumerates valence and sea quark components of the nucleon and the effective charge $\tilde{Q}_i$, which should be replaced by $\widetilde{C}_V$ (for vector current operator) and $\widetilde{C}_A$ (for axial vector operator) in the neutrino DIS on a nucleon/nucleus, is given in Table II. The parameter $0 \le \alpha_i \le 1$ in Table II characterizes the strength of the superconducting phase.

In case of strong superconductivity, which means some of the α_i is 0, $F_2^N(x) \sim F_2^P(x)$. The experimental measurement of $F_2^P(x)$ and $F_2^N(x)$ in the deep inelastic scattering experiments shows such a tendency manifests in the violation of Gottfried sum rule [49]. Perturbatively, the effects due to various flavor violation is too small to account for the magnitude of violation. One of the plausible interpretation is that there is a flavor violation in the sea [50] due to mesonic (pion and heavier mesons) excitations. The latest rather complete study (e.g. Szezurek and Speth in Ref. [50]) still can not explain the violation. If we use the normal electrical charges for the up and down quarks, then there is a reduction of $f_i(x)$, which is interpreted as a violation of the Gottfried sum rule.

In a large nucleus, the effects of diquark condensation are expected to be enhanced due to the increase of the baryon density and the size of the system; this in general produces the depletion of quark number [51] observed in the EMC effects in a nucleus, which is related to a (enhanced) violation of Gottfried sum rule in the picture given here, since the antiquark components in a nucleus is known to be small in experimental observations [52,53]. Fermi motion and binding effects are not enough to explain the EMC effects if proper normalization of baryon number is carried out [54] without rescaling. It is generally accepted that the EMC effects are still not well understood [55].

One of the decisive experiments, which can be used to test the picture proposed for the violation of Gottfried sum rule and the EMC effects in this paper, is the use of , instead of the charged lepton and nucleon/nucleus DIS, the neutrino and nucleon/nucleus DIS. This is because the neutrino couples only to the hadronic neutral current, which has smaller isoscaler component of the vector current (see Table II). If the picture given here is true, then the magnitude of the violation of the Gottfried sum rule and the EMC effects should be reduced in the neutrino DIS on nucleon/nucleus.

The effective charges for the $\bar{l} + l \to hadrons$ production processes are also given in Table II. They are not affected even if the hadrons produced in the final states contains diquark condensation. Since the quark-antiquark pair, which hadronizes into hadrons in the final state, are current quarks created in the vacuum within short time period in the collision and no diquark condensate is believed to be exist in the vacuum at the present day conditions because it carries color.

VIII. SUMMARY

A natural way of extending the PCAC relation beyond the conventional one is presented in this study. For a relativistic fermionic system, the extension presented here is unique if assumptions I, II are valid. They are partially supported by the phenomenology. The fact that there can only be two kinds of spontaneous chiral symmetry breaking phases in nature that preserve flavor symmetry considerably reduces the number of possibilities that one should consider.

A detailed study of the effects of these chiral soft longitudinal modes and that of spreading of the isoscalar part of the charge of constituent quarks depends upon a more specific model for a nucleon. It is beyond the scope of this paper, despite the fact that this study put strong constraints on any such an attemp. It is an interesting topic to be further explored.

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FIG. 1. The ϕ^2 dependence of a_δ. Λ is the chiral symmetry breaking scale which is chosen to be the value of the covariant cut off in the quark loop integral. In order to obtain a smooth curve, the sharp cut off in the Euclidean momentum space integration is replaced by a smooth one. The shape of the smooth cut off $F(p/\phi)$ is plotted on the same graph with an arbitrarily chosen ϕ.

FIG. 2. The ϕ^2 dependence of the quark-diquark coupling constant $g_{\delta q}$. The same smooth Euclidean momentum space cut off as the one used in Fig. 1 is used.

FIG. 3. The ϕ^2 dependence of the Goldstone diquark decay constant f_δ. The same smooth Euclidean momentum space cut off as the one used in Fig. 1 is used.

FIG. 4. The $\rho\pi$ reduced density of states in the pionic channel as a function of q^2 obtained from the fitting procedure for two values of the $\rho\pi\pi$ coupling constant. For comparison, the one loop ($\lambda = 0$) reduced density of state is drawn with a dashed line.

TABLE I. The results of fitting, where $g_A = 1.26$, $g_{\pi NN} = 13.4$, $f_\pi = 93.2$ MeV, $q^2_{min} = -0.6$ GeV^2 and $q^2_{max} = 0.2$ GeV^2. The unit for all but λ and $g_{\rho\pi\pi}$ is GeV. λ and $g_{\rho\pi\pi}$ are dimensionless. The quantities with a star on top is chosen by physical considerations.

$g_{\rho\pi\pi}/4\pi$	$\Lambda_{\pi NN}$	λ	$\sqrt{I_B}$	$\sqrt{s^*_R}$	M^*_A	$\sqrt{s^*_{th}}$
1.0	0.94	1.12	1.12	1.30	1.00	0.91
1.5	0.93	0.41	1.26	1.30	1.00	0.91
2.0	0.91	-0.0023	1.34	1.30	1.00	0.91
2.5	0.89	-0.25	1.27	1.30	1.00	0.91

2.9	0.88	-0.39	1.30	1.30	1.00	0.91
3.5	0.86	-0.56	1.35	1.30	1.00	0.91

TABLE II. The effective charges for the electro-weak couplings in the DIS between leptons and a nucleon/nucleus and in lepton-antilepton production of hadrons according to the standard model. $\tilde{C}_V$ and $\tilde{C}_A$ are coefficients of the vector and axial vector current operators in the hadronic weak neutral current operator. Here θ_W is the Weinberg angle, "hs" denotes hadrons, "u" and "d" denotes up and down quark respectively. The corresponding charge for an anti-quark is just opposite. The value for α in the table depends on the color and the momentum fraction x of the corresponding quark if the flavor symmetry is preserved.

Quark	$\tilde{Q}(l+h \to l+h)$	$\tilde{C}_A(\nu+h \to \nu+h)$	$\tilde{C}_V(\nu+h \to \nu+h)$
u	$\alpha\frac{1}{6}+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}-(\frac{\alpha}{3}+1)sin^2\theta_W$
d	$\alpha\frac{1}{6}-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}-(\frac{\alpha}{3}-1)sin^2\theta_W$
Quark	$Q(\bar{l}+l \to \overline{hs}+hs)$	$C_A(\bar{\nu}+\nu \to \overline{hs}+hs)$	$C_V(\bar{\nu}+\nu \to \overline{hs}+hs)$
u	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}-\frac{4}{3}sin^2\theta_W$
d	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}+\frac{2}{3}sin^2\theta_W$